



Lesson Plan for Implementing NETS•S—Template I (More Directed Learning Activities)

Template with guiding questions

Teacher(s)

Name

Shanna Irving & [REDACTED]

Position

Teachers

School/District

[REDACTED] High School/[REDACTED] School District

E-mail

Smc3672@students.kennesaw.edu

Phone

7708157395

Grade

Level(s)

11

Content Area

Algebra with Math Support

Time line

1.5 hours per class per day; 2 classes; 2 days

Standards

Make sense of problems and persevere in solving them.

- Students will **make sense of concepts** and demonstrate their ability to solve relevant math problems on the Google Doc.

Reason abstractly and quantitatively.

- Students will apply oral language and visual mnemonic links to mathematical concepts.

Construct viable arguments and critique the reasoning of others.

- During collaboration, students will **critically evaluate** mems and resources before selecting them to include in the Google Doc. They will argue for or against resources and defend their positions.

Model with mathematics.

- A column on the Google Doc is dedicated to students' demonstrations of how to mathematically employ each concept; these demonstrations act as models for other students in the course.

Use appropriate tools strategically.

- Students will **use both mathematical and technological tools** effectively in order to complete their portions of the Google Doc and to thus become reciprocal teachers.

Skill Standards

ISTE*S 1: Students will use the existing and developing knowledge to **generate engaging mnemonic mems** for each reviewed concept. They will collaborate to determine the real-world applicability of each math concept and hyperlink evidence of that applicability via online tutorials and games that will mimic possible Unit 3: Polynomials & Asymptotes summative assessment content.

ISTE*S 2: Students will **collaborate with teacher-selected partners** to generate a whole class collaborative study guide for their Unit 3: Polynomials & Asymptotes summative assessment. They will practice communicating verbally with partners and digitally via text, images, videos, and other hyperlinked internet content with the whole class in order to solve a dual problem: (1) work together using the concept of reciprocal teaching to prepare for the Unit 3: Polynomials & Asymptotes summative assessment, and (2) understand how math is relevant to the real world.

ISTE*S 3: Students will collaboratively and carefully select images for mnemonic mems and **Internet content for real-world applicability** of assigned math concepts. They will search for videos and games that demonstrate and provide practice with the concepts effectively, evaluating each before hyperlinking in the study guide.

ISTE*S 4: Students partnerships will determine how to investigate the math concepts' real world applicability. Partners will also determine how best to **manage the workload** during collection and evaluation of math concept-related Internet content, and they will invite feedback from their diverse group of peers upon completion of their portion of the study guide.

ISTE*S 5: Students will learn to hyperlink all Internet content and will **collaborate positively and responsibly**. Their whole class efforts will produce a useful tool that will increase their appreciation for digital collaboration and will demonstrate their digital leadership abilities.

ISTE*S 6: Students will understand and use the course blog, Google Doc, Memrise.com, screencasts, Snipping Tool, and hyperlinks to digital content they locate through Internet research. They will **work collaboratively to resolve problems** with technology and to use the technology tools presented flexibly to benefit novel digital circumstances.

[NETS*S Standards:](#)

Overview

1. **Discussion:** Who is comfortable with technology? Who uses technology for educational purposes? What are some ways? Apps you use? Who would like to learn about a tool that will help you to study vocabulary and course content more effectively and in an engaging way?
2. **Present Primary Tool:** Play Memrise screencast video and share link on board so students can access as needed. Students create Memrise account.
3. **Present Tools 2, 3, & 4:** Students are directed to teacher's blog, then to Google Doc link. They will be walked through use of the Snipping Tool and how to hyperlink and will be directed to the screencasts, linked within the Google Doc and on the teacher blog, that reiterate the processes.
4. **Present assignment:** Students are paired and assigned one mathematical concept from the Google Doc study guide for their upcoming assessment which covers content students have found difficult. Each pair of students accesses the Memrise course created by Mr. [REDACTED] and me, and, using those terms/concepts, then begins practicing creating mems, studying terms.
 - **STEPS:**
 - **First**, the partners collaborate to create mems to help them remember and understand the terms/concepts present in the Memrise course.
 - **Second**, they are given one term/concept for which they are to be the reciprocal teachers.
 - **Third**, the partners evaluate all the mems created by their classmates for the assigned term they will teach. They copy and paste that mem into the shared Google Doc.
 - **Fourth**, the partners re-word the definition of their assigned term and insert the re-wording into the shared Google Doc.
 - **Fifth**, the partners use the Internet to search for information that demonstrates real-world applicability of the concepts. They insert these into the Google Doc using hyperlinked text.
 - **Finally**, the partners write and work a problem that employs that mathematical concept, take a picture of the problem, and upload it into the Google Doc as a model.
5. **Students Get to Work for Predominance of Class Periods:** With teachers' facilitation and technological and content-explanation guidance, the partners follow the steps outlined above.
6. **Teachers Assess:** (Formative) Ongoing assessment of content creation during activity via digital monitoring and commenting as well as individual discussions with students during room monitoring. (Summative: See Appendix C) Students will be assessed on overall demonstrated understanding of math concepts, overall creativity with content, depth and clarity of content-real world applicability links, and correctness of worked problems as evidenced by their piece of the shared Google Doc. (Summative: See Appendix D) Students will be assessed on their mathematical skill via the Unit 3: Polynomials & Asymptotes test.

Essential Questions

Essential Questions:

- How do we demonstrate math concepts creatively?
- How are these math concepts relevant in the real world?
- How do we learn how to learn?

Guiding Questions:

- How can you relate that concept in your own words?
- What kind of funny, sad, etc. ideas can be associated with this math concept?
- What might be a good search term for that association?
- How can you integrate the humor, etc. with the concept terminology into the mem text?
- What Google search terms might be most appropriate to find related games, videos, or tutorials?

Prior Knowledge:

- Students will have some working knowledge of these concepts, having learned and practiced working with them during previous unit. They should thus be able to relate the concepts in manageable terms after working with the concepts through Memrise quizzing and mem creation. Students who need to review in more depth and for a longer time will have complete accessibility in order to work on their portions of the assignment at home, in the library, or elsewhere before and after school hours. Students with smart phones have the option of downloading and practicing the course via Memrise app.

Expected Knowledge Development:

- Students will create mnemonic mems for all terms to be assessed. The mems require creativity and depth of content understanding. Students will become the teachers of one of the terms. Each partnership will evaluate its classmates' and its own mems for that term in order to choose one mem for the collaborative Google Doc. Students will then generate re-worded definitions of the terms, demonstrating their thorough comprehension. Furthering their content knowledge, each partnership will use the Internet to research its assigned concept in order to find real-world applicability, videos, games, and other digital resources that will help demonstrate the concept fully. Finally, they will create a model problem to upload, demonstrating their ability to apply the concept numerically. Ultimately, they will use their assigned portions to digitally and reciprocally teach their classmates via engaging mems, student-worded definitions, concept application hyperlinks, and problem models.
- This should result in deeper learning of the math concepts as evidenced by improvements in the Unit 3: Polynomials & Asymptotes test scores.

Assessment You must attach copies of your assessment and/or rubrics. Include these in your presentation as well.

Products

Students will produce mems for each term they are studying, and they will evaluate and select the best mem for their assigned study guide concept. They will post the mem into an open, shared Google Doc hyperlinked to Mr. [REDACTED]'s blog. This Google Doc will demonstrate their abilities to relate visually memorable mnemonic content, explain concepts in their own words, locate and evaluate proof of real-world applicability, and complete model problems.

- Google Doc Link: <http://tinyurl.com/kphm98y>

Assessment

- Students will be assessed formatively via ongoing assessment of content creation during activity. Digital monitoring and commenting through the Google Doc as well as individual discussions with students during room monitoring will determine development of mastery.
- Students will be assessed formatively via determination of demonstrated understanding of all mems created, mem selected for assigned math concept, accuracy of definition re-wording, creativity and relevance of content hyperlinks, depth and clarity of content-real world application links, and proper hyperlinking to give digital credit to sources.
- Students will be assessed summatively on the Unit 3: Polynomials & Asymptotes assessment, which is not included in the time frame for this lesson.

Differentiation

- Assessments will be differentiated by content via strategic grouping of partners and assignment of math concept, by process via teacher-selected partnerships and individualized assistive technology, and by product via partnership choice of hyperlinked or partnership-generated and uploaded content for reciprocal teaching.

Assistive Technology

- In this math class, the student population includes four learners that benefit from assistive technology. Two students have learning disabilities (LD), and two are autistic (AU). Mr. [REDACTED] reports that the students with LD struggle with dyslexia specifically. To assist them, I will provide them with a link to YakItToMe.com, a website that converts text to speech. He also reports that the students with AU work well with certain partners only, and they will thus be assigned those partners.
- Several items were built into the lesson plan specifically to support students who struggle to pay attention and remain engaged during whole class instruction. The screencast videos ensure that these students can access the information at will. The use of game-based learning and research on games is meant to support students who become disengaged with purely academic approaches to learning.
- The screencasts also help students with auditory processing difficulties by providing visual step-by-step instructions they can access and pause at will.
- The shared Google Doc is a graphic organizer that helps students understand and practice with the many facets associated with truly learning a math concept.

Resources

Digital Tools & Student Learning

Screencasts: Memrise, Snipping Tool, & How to Hyperlink

- Students will be presented with these tools via overviews of how to use them. They will then have access to these screencasts in order to review how to use them as needed. This accessibility provides students with the ability to learn at their own paces.

[Memrise.com](#)

- Memrise.com is a website and mobile application that offers free access to thousands of courses and mnemonic memes (mems, as the Memrise team named them) generated by staff and users. These courses can help students learn other languages and other curricular content at will. Student partnerships will generate Memrise courses based on their assigned math concepts and invite their other classmates to practice their courses as well.

[Google Doc](#)

- Students will demonstrate their knowledge on a whole-class shared Google Doc. They will learn to navigate the Google Drive and to use it with peers to engage in collaborative learning and content sharing.

Snipping Tool

- Students using desktop computers will have access to the Snipping Tool that can cut the pictures out of Memrise and put them in the Google Doc. Students on student laptops will not have Snipping Tool functionality. For students who cannot access Snipping Tool for this or any other reason, I will demonstrate how to copy and paste into a Google Doc using the CTRL+C and CTRL+V.

[Yak It to Me](#)

- Students with dyslexia or difficulties reading on-screen text can use this free program to read text aloud to them. Students with autism can also use this program to help them with verbal communication.

[Google Forms Survey](#)

- Students will have the option of completing the student survey on paper or by using their phones and accessing this Google Forms survey, which will convert the answers to spreadsheet data Mr. [REDACTED] and I can use to inform our future implementations and to support the integration of this lesson into other classes.

Previous Technology Skills & Knowledge

Typing & Navigation Skills

- Students will need to type information in appropriate areas.

Instructional Plan

Preparation

Needs

- Students are approaching a summative assessment for Unit 3: Polynomials & Asymptotes and are in need of effective study strategies and study tools. They have met with difficulty in this unit.
- I discovered this via conversation with their teacher, Mr. [REDACTED]

Interests

- Students will explore the math concepts in search of interesting and engaging images based on their interests.
- These interests are made salient through the mem creation.
 - For example, two male students who are interested in military careers created mems that reflect that interest by including such imagery as a drill sergeant screaming at one troop in a line of troops. They used this to image to relate the idea of Least Common Denominator (LCD), because the troop receiving the verbal abuse must have done something to make him the LCD in the line.

Prior Learning

- Students have already worked with each of these math concepts throughout the semester and will be generating a review for an upcoming summative assessment.
- I discovered this via conversation with their teacher, Mr. [REDACTED]

Possible Difficulties

- Students sometimes have difficulty re-wording definitions, but this step is essential to truly considering the meaning of a concept.
- Students may have difficulty with the student laptop computers' hardware and software. The five year old laptops have seen misuse and sometimes have battery longevity weaknesses.
- Because I am using another teacher's classroom, my own lack of familiarity with the setup may become an obstacle.

Management

Management Strategies

- During direct instruction: To ensure that students are paying attention and completing steps as I demonstrate them, I will situate partnerships in paired desks facing the interactive whiteboard. While I deliver instruction, the classroom teacher, Mr. [REDACTED] will circulate, observe, and assist by remaining behind the desks where he can easily view the laptops.
- During assignment facilitation: Mr. [REDACTED] and I will circulate, observe, and assist. We will also take turns at a computer providing content feedback to groups via the Google Doc's comments option.

Equitable Access Strategies

- Students will be provided with laptops, each with access to the Internet and, via Mr. [REDACTED]'s blog, to all of the digital tools to be used in the lesson.
- Students with LD and AU will be presented with the program Yak It to Me, a text-to-speech converter.

Troubleshooting Plan

- Before: All laptops will be fully charged and updated and the projector and interactive whiteboard will be checked for functionality before class begins. A full rehearsal of the lesson on the teacher's computer/interactive whiteboard and on a student laptop will be completed prior to demonstrating the lesson to students.
- During: Extra laptops will be on hand to ensure that any malfunctioning laptops can be quickly replaced. If the Google Doc experiences lag with so many students on it at once, students will be directed to fill in a Word document they can download from Mr. [REDACTED]'s blog.

Instructional Strategies and Learning Activities

Research-based Learning

- Reciprocal Teaching & Definition Re-Wording:
 - In mathematics, reciprocal teaching has demonstrated promise when augmented to a curriculum that emphasizes making predictions, clarifying content and relationships, asking questions, and generating summaries of information interconnectedness (Meyer, 2014). This assignment asks students to complete each of these per concept and adds the element of reciprocal teaching by dividing the content load up by student partnerships and providing the entire class with access to the generated materials.
- Elaborate Encoding/ Multifaceted & Repeated Content Exploration:
 - Memories are generated via encoding information with prior knowledge; the depth of the encoding allows for better retention of information (Galli, Bartrés-Faz, & Craik, 2014). This assignment uses Memrise, a tool that presents students with concepts in multiple ways (visual, creative, definition first, word first, spelling), repeating concepts strategically and gradually adding more as it assesses students' readiness.
- Collaborative Learning of Vocabulary Concepts:
 - Improved vocabulary development and retention has been linked to small group work (Dobao, 2014). This lesson involves two tiers of student collaborative learning: (1) a direct face-to-face tier with partners; (2) an indirect whole-class collaboration via the Google Doc.

Higher Order Thinking (Bloom's Taxonomy)

- Remembering: Students will remember the content presented to them during the previous unit.
- Comprehending: Students will demonstrate comprehension on content by generating their own wording for definitions.
- Analyzing: Students will demonstrate their analysis by creating mnemonic links to the concepts.
- Applying: Students will apply their understanding by including a worked math problem for each concept.
- Evaluating: Students will collaboratively evaluate mems and website resources to include in the shared Google Doc.

Relevance to Teaching & Learning

- Presenting students with methods to attain depth of thinking about each relevant concept associated with a course enhances student learning. Students will develop a depth of knowledge about the concepts assigned to them and have review materials ever-available to them via the Internet for the remaining concepts on the upcoming Unit 3: Polynomials & Asymptotes assessment. This ensures that students enter the second half of the course with complete and thorough prior knowledge, a necessity to success in the second half.

Communication & Collaboration

- Students will participate in two levels of collaboration, including direct collaboration with a partner and indirect collaboration with the whole class, and multiple modes of communication, including oral and digital.

Differentiation

Differentiating Content

- Student partnerships will be assigned terms and concepts to explore. The allocated terms' difficulty will take students' strengths and weaknesses into account.

Differentiating Process

- Some students will need extra assistance reading and writing text on the screen; these students will be presented with Yak It to Me.
- The Google Doc is designed as a graphic organizer that helps students with executive functioning.
- All students will receive individualized momentary-need assistance digitally and interpersonally. Process changes will be flexible dependent on students' needs; students who struggle to generate mnemonic links, for example, will be provided with on-the-spot scaffolding via inquiry that will lead them to the creation of the mnemonic link.
- Students will have the option of completing the student survey on paper or using the digital [Google Forms survey](#).

Collaborative and Independent Learning

- The collaborative requirement during creation of mems and completion of each partnership's portion of the study guide ensures collaborative learning. Interactions with the students will facilitate the collaboration by encouraging co-construction of knowledge and mediating any conflicts.
- Independent learning will be encouraged as a means of studying for the Unit 3: Polynomials & Asymptotes summative assessment and can be accomplished from anywhere with Internet access using the Memrise course as well as the filled Google Doc.
- Students will be provided with a class-generated study tool that presents each relevant concept in a multifaceted way and encourages depth of exploration for each.

Opportunities for Enrichment

- Students who finish early or do not find this process challenging will be provided with the opportunity to create their own educational resources instead of simply locating them and hyperlinking them into the appropriate elements of the document.

Reflection

Closing Event

- The closing event was a class discussion about the lesson. In my absence, which ensured honest discussion, Mr. [REDACTED] distributed the simple survey attached as Appendix B to students that did not have their phones. For students who preferred completing the surveys digitally, he provided them with a hyperlink to [this Google Forms survey](#).

Lesson Effectiveness

- Students gradually increased their ability to think analytically about the math concepts present in the Memrise course. The new approach to learning math made them hesitant at first, but most partnerships developed interest in working creatively, quickly, and accurately once they felt comfortable with the assignment format and requirements and they noticed the gaming aspect of earning points as projected on the Memrise leaderboard for the course. With more time allotted by Mr. [REDACTED], it is projected that they could have completed more than just the mems and definitions by the end of the two class periods.
- Ultimately, the lesson's effectiveness is in its scaffolded development of critical thinking skills and its usefulness across the curriculum.

Ideas for Improvement

- Presenting the ultimate goal, the completed Google Doc, to the students first was too much, too soon. They were lost in all my explanation. A more effective approach for next time will be to teach Memrise first, then explain the Google Doc and what they will do with their new depth of knowledge. Baby steps.
- I did not consider that some students may not have an email address by 11th grade. One partnership had no email address. Luckily, Mr. [REDACTED] was able to work with them individually to set up email accounts quickly and catch back up to the rest of the class. In the future, I will include in the lesson preparation a discussion about email accounts and time to generate them before the date of the lesson.
- During my rehearsal on a student laptop, I did not attempt to create a new account. I logged into my account and searched for the course to ensure that the students would be able to find out and that I could direct them to it smoothly. The website developers have added new hoops to jump through once users sign up, and I was unprepared to visually present how to jump through those hoops. In the future, I will create a throwaway account or have a student log in using the projected computer so students can watch the navigation real-time while they navigate as well.
- Because I watched so many students' brains beginning to open analytical channels and become comfortable with doing so, I can see that this lesson would be more powerful for the development of lasting, impactful critical thinking skills if introduced at the beginning of the semester and repeated throughout the semester with new content.
- Three students became disengaged: one, after the lunch break that divides the class period, looked like she had been crying and put her head down and slept. Her partner was left alone and another student came in late and had no partner, so I put them together. They would not communicate but acted like they would when I spoke with them intermittently. This partnership ultimately generated two mems after long coaxing but went no further. Mr. [REDACTED] stated that the lack of effort they presented is consistent with the lack of effort they present on other assignments in all classes. Research suggesting that integration of technology and authenticity would motivate at-risk students did not hold true for them, and this deserves further exploration.

Closure:

Lessons Learned

- I noticed that students had a tendency to spend more time on their phones than on the laptops. Providing them with technology to use, then, may not decrease the amount of time they spend distracted by technology used for non-educational purposes. I began to find it necessary to have them put their phones away, as the phones were being used for texting and music, not completing the lesson.

Advice

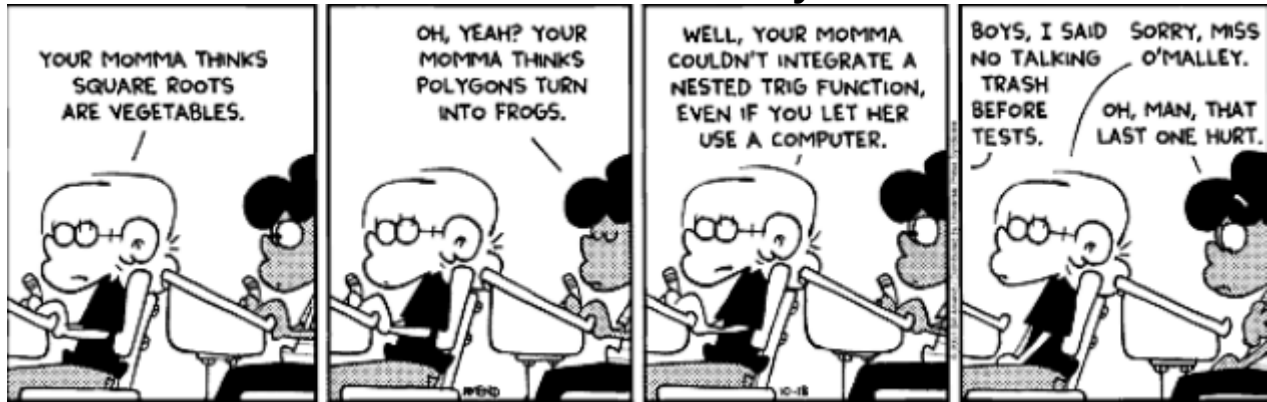
- Introduce tools early in the course and repeat it throughout so students have time to develop critical thinking skills associated with the curriculum.
- Require that students put away their personal devices when provided with laptops or iPads.
- Be strategic with desk placement, especially if you do not have a co-teacher who can watch the screens from the back of the room and help students who lag behind in steps while you present information at the front of the room using the interactive whiteboard.
- Introduce Memrise first. Only introduce the ultimate Google Doc assignment once students are comfortable with the concepts in the Memrise course and have plenty of partnership-generated mems to choose from.
- Walk students through creation of one mem in the course you assign them. Do it on the projector while they do it on the laptops. Discuss the significance of using the word or a related word on the picture when generating the mnemonic connection between the image, the concept, and the concept's definition.
- Be prepared to help struggling students make mnemonic links at first and gradually release the responsibility as they build confidence.

References

- Dobao, A. F. (2014). Vocabulary learning in collaborative tasks: A comparison of pair and small group work. *Language Teaching Research*, 18(4), 497-520. doi:10.1177/1362168813519730
- Galli, G., Bartrés-Faz, D., & Craik, F. (2014). What makes deeply encoded items memorable? Insights into the levels of processing framework from neuroimaging and neuromodulation. *Frontiers In Psychiatry*, 51-8. doi:10.3389/fpsy.2014.00061
- Meyer, K. (2014). Making meaning in mathematics problem-solving using the Reciprocal Teaching approach. *Australian Journal Of Language & Literacy*, 37(2), 7-14.

Appendices
Screencast Review of Lesson Video (Embedded Link)

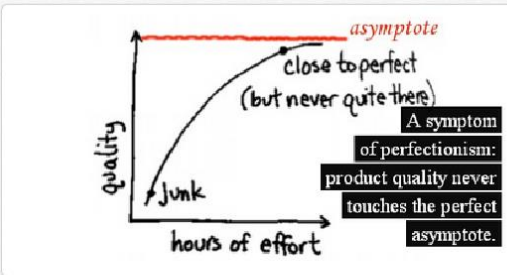
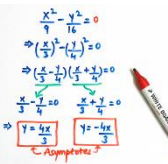
Math Final Exam Study Guide



CLICK HERE for a list of Student User-Generated Mem Courses for this Content.

Follow each course on Memrise.

Use the courses and this document to study for the Unit 3 test!

Math Term or Concept	Your Original Mem Memrise Tutorial Snipping Tool Tutorial (<u>alternative</u> : Just copy & paste using CTRL+C and CTRL+V key strokes) Hyperlinking Tutorial	Definition & Explanation in Your Own Words	Concept Relevance in the Real World Website, Image, Game, & Video Links	Example Worked Problem
Asymptote	<p style="text-align: center;">You have chosen this mem:</p> 	<p>line that can be drawn beside the curve; the line and the curve's side run side by side to infinity and never touch but come consistently closer together</p>	<p>How to find asymptotes: Vertical and horizontal asymptotes of rational functions</p> <p>Real world example: Average cost of sale price ice cream</p> <p>Practice game: Math is Fun</p>	<p>Click below to see example full-screen:</p> 

Rational Function				
LCD				
Rational Equations				
Adding Rationals				
Subtracting Rationals				

Excluded Value				
Function				
Vertical Asymptote				
Horizontal Asymptote				
Domain				

Range				
Simplify				
End Behavior				
Extraneous Solution				

Appendix B
Student Survey

<ul style="list-style-type: none">• Did you find the lesson meaningful and worth completing?• Do you think the tools are useful?• Would you use them again? <p><u>Comments</u></p>	<ul style="list-style-type: none">• Did you find the lesson meaningful and worth completing?• Do you think the tools are useful?• Would you use them again? <p><u>Comments</u></p>
<ul style="list-style-type: none">• Did you find the lesson meaningful and worth completing?• Do you think the tools are useful?• Would you use them again? <p><u>Comments</u></p>	<ul style="list-style-type: none">• Did you find the lesson meaningful and worth completing?• Do you think the tools are useful?• Would you use them again? <p><u>Comments</u></p>
<ul style="list-style-type: none">• Did you find the lesson meaningful and worth completing?• Do you think the tools are useful?• Would you use them again? <p><u>Comments</u></p>	<ul style="list-style-type: none">• Did you find the lesson meaningful and worth completing?• Do you think the tools are useful?• Would you use them again? <p><u>Comments</u></p>

Appendix C
Completed Google Doc Rubric

STUDENT NAME: _____

DATE RECEIVED: _____

Unit 3 Review Study Guide Entry Rubric

Criteria	Lacking 15	Adequate & Complete 20	Thorough & Creative 25
overall demonstrated understanding of math concepts	Mem and reworded definition do not reflect understanding of the assigned concept	Mem and reworded definition reflects adequate understanding of the assigned concept	Mem and reworded definition reflects deep understanding of the assigned concept
overall creativity with content	Mem and hyperlinks do not reflect creativity with the assigned concept	Mem and hyperlinks reflects adequate creativity with the assigned concept	Mem and hyperlinks reflects creativity with the assigned concept
depth and clarity of content-real world applicability links	Real world connection and hyperlinks are faulty or not present	Real world connection and hyperlinks relate directly to the concept and help demonstrate the real-world relevance	Real world connection and hyperlinks demonstrate the concept in thorough, engaging ways that stress the real-world relevance of the concept
correctness of worked problem	The worked problem is not available or is not worked correctly.	The worked problem is available and correct.	The worked problem is available, correct, thorough, and clear as a teaching tool for other students.

partner assessment partner's name: _____	My partner was not helpful.	My partner was somewhat helpful.	My partner was helpful and creative.
---	-----------------------------	----------------------------------	--------------------------------------

Appendix D

Unit 3 Summative Assessment: Polynomials & Asymptotes (version partially and incorrectly completed by student prior to lesson implementation)

Advanced Algebra with Support Name: _____
DCC, 2, 2014
 REVIEW TEST 1, UNIT 3

1. Multiply:

a) $\frac{1}{x-4} \cdot \frac{4x-16}{5x^2-20x}$
 $\frac{1}{x-4} \cdot \frac{4(x-4)}{5x^2-20x}$
 $\frac{4}{5x^2-20x}$

b) $\frac{2}{x-9} \cdot \frac{x^2-x-72}{x+8}$
 $\frac{2}{x-9} \cdot \frac{x+8(x-9)}{x+8} = 2$

2. Divide:

a) $\frac{21n+14}{7} \div \frac{18n^2+12n}{2n}$
 $\frac{21n+14}{7} \times \frac{2n}{18n^2+12n}$
 $\frac{21n+14}{7} \times \frac{2n}{6n(3n+2)}$
 $\frac{7(3n+2)}{7} \cdot \frac{2n}{6n}$
 $\frac{2n}{6n} = \frac{1}{3}$

b) $\frac{9}{n-1} \div \frac{n^2+17n+72}{n^2+8n-9}$
 $\frac{9}{n-1} \times \frac{n^2+8n-9}{n^2+17n+72}$
 $\frac{9}{n-1} \times \frac{(n+9)(n-1)}{(n+9)(n+8)}$
 $\frac{9}{n+8}$

3. Add:

a) $\frac{n-3}{n^2-10n+25} + \frac{4n+3}{n^2-10n+25}$
 $\frac{5n}{n^2-10n+25}$

b) $\frac{2v}{5} + \frac{4v}{v-5}$

4. Subtract:

a) $\frac{n+5}{10n^2-2n-8} - \frac{n-2}{10n^2-2n-8}$
 $\frac{3}{10n^2-2n-8}$
 $\frac{3}{10n^2(x-4)(x+2)}$

b) $\frac{6r}{r+1} - \frac{4}{r-2}$

$n < m = y = 0$
 $n = m = y = a/b$
 $n > m = \text{no solution}$

5. Simplify and find the excluded value:

a) $\frac{28a^2 - 24a}{12a}$

$$\frac{\cancel{4}a(7a-6)}{12\cancel{a}} \quad | \quad 12a \neq 0$$

$$\frac{7a-6}{3} \quad | \quad a \neq 0$$

b) $\frac{63n}{21n+70}$

$$\frac{9\cancel{n} \cdot 7}{7(3n+10)} = \frac{9n}{3n+10}$$

$$\frac{21n+70 \neq 0}{21} = \frac{-70}{21}$$

$$n = \frac{-70}{21} \neq \boxed{\frac{-10}{3}}$$

6. For each function find the vertical and horizontal asymptotes and identify the domain and range.

a) $f(x) = \frac{1}{x^2 - 2x - 3}$

$$(x-3)(x+1) = 0$$

$$x-3=0 \quad x+1=0$$

$$x=3 \quad x=-1$$

VA $x=3, x=-1$

Domain $\mathbb{R}, x \neq 3, x \neq -1$

HA $y=0$

Range $\mathbb{R}, y \neq 0$

b) $f(x) = \frac{3}{x+4}$

$$x+4=0$$

$$x=-4$$

VA $x=-4$

Domain $\mathbb{R}, x \neq -4$

HA $y=0$

Range $\mathbb{R}, y \neq 0$

c) $f(x) = \frac{x^2 - x - 12}{x^2 - 9}$

$$x^2 - 9 = 0$$

$$+9 \quad +9$$

$$\sqrt{x^2 - 9} = \sqrt{9}$$

VA $x = \pm 3$

Domain $x \neq 3, x \neq -3$

HA $y = \frac{1}{1} = 1$

Range $\mathbb{R}, y \neq 1$

d) $f(x) = \frac{x^2 + x - 12}{-3x}$

$$-3x = 0$$

$$x = 0$$

VA $x = 0$

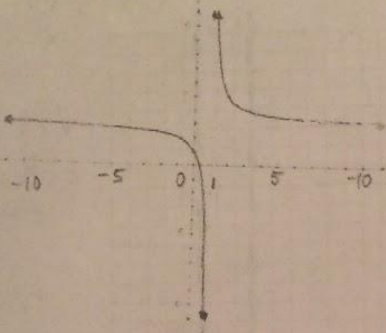
Domain $\mathbb{R}, x \neq 0$

HA no solution

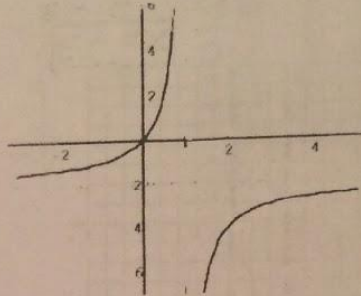
Range \mathbb{R}

7. Draw vertical and horizontal (if there are any) asymptotes through the graph and write the equation of the lines.

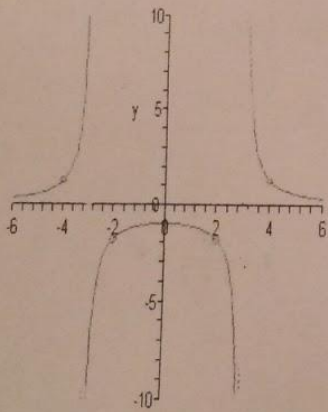
a)



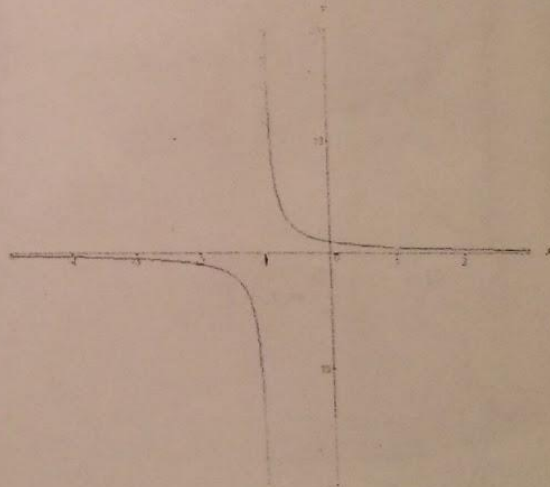
b)



c)



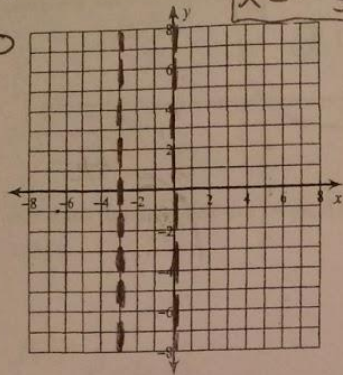
d)



8. Find the vertical and horizontal asymptotes of the function and draw them on the coordinate plane. Then, using calculator, sketch the direction of the graph.

a) $f(x) = \frac{5}{x+3}$ $x+3=0$
 $-3 \quad -3$
 $x = -3$

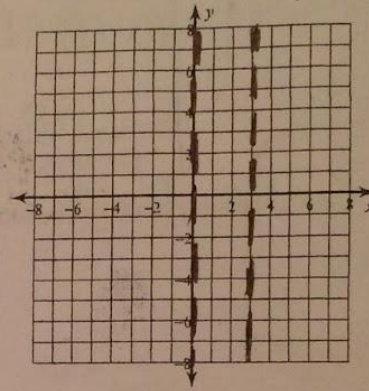
VA = $x = -3$
 HA = $y = 0$



b)

$f(x) = \frac{x}{x^2-9}$

$x^2 - 9 = 0$
 $+9 \quad +9$
 $\sqrt{x^2} = \sqrt{9}$
 $x = 3$



VA = 3
 HA = $y = 0$

9. Solve for x. Check for extraneous solutions.

a)

$\frac{32}{x} = 2x$

b)

$\frac{9}{x-1} = \frac{3}{2}$

c)

$\frac{15}{x} = x + 2$

d)

$4 + \frac{6}{x-2} = 5$